SHRI ANGALAMMAN COLLEGE OF ENGINEERING AND TECHNOLOGY
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MA 1251- NUMERICAL METHODS

FIFTH SEM. ECE/IT

Unit-I: SOLUTION OF EQUATIONS AND EIGEN VALUE PROBLEMS

PART-A

1. Define algebraic and transcendental equation
2. State the iterative formula for method of false position to solve f(x)=0
3. If f(x) = 0 has root between x=a and x=b, then write the first approximation root by the method of false position
4. Find the approximate real root of $xe^x - 3 = 0$ in $1<x<1.1$ by method of false position.
5. State the order of convergence and convergence condition for Newton's method.
6. Write the iterative formula for finding $\sqrt{N}$, where N is a real number, by Newton's method
7. On what type of equations Newton's method can be applicable?
8. State the condition and order for convergence of f(x) = 0 using Newton-Raphson method.
9. What is the criterion for the convergence of Newton-Raphsons method?
10. what is the sufficient condition for the convergence in iteration method?
11. State the order of convergence and convergence condition for Newton's method
12. What is the condition for the convergence of Gauss-Jacobi method is solving a system of simultaneous linear equations.
13. Why Gauss Seidel iteration is a method of successive corrections?
14. Write down the condition for the convergence of Gauss-Seidel iteration scheme.
15. State the sufficient condition on $\phi(x)$ for the convergence of an iterative method for f(x) = 0 written as $x = \phi(x)$

16. Compare Gausian elimination and Gauss-Jordan methods in solvingthe linear system [A] {X} = [B]
17. Solve the system of equations x-2y = 0, 2x + y = 5 by Gaussian elimination method
18. Solve by Gauss elimination method 2x+y=4; x+2y=5
19. By Gauss elimination method solve x+y = 2 & 2x+3y = 5
20. Solve 2x + y = 3 , 7x - 3y = 4 by Gauss elimination method
21. find inverse of $A = \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$ by Gauss-Jordan method
22. State any two difference between direct and iterative methods for solving system of equations
23. Define Eigen Value and Eigen vector
24. How do you find numerically smallest eigen value of a matrix A?
25. Find the dominant eigen value of $A=\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ by power method.

PART-B

1. Find the real root of the equation $x \log_{10} x - 1.2 = 0$ correct to four places of decimal using false position method.
2. Find the root which lies between 2 and 3 corrrect to e places of decimals of $x^3 - 5x - 7 = 0$, using the method of false position
3. Find the real root of $x^3 + x^2 - 100 = 0$ by using iteration method .
4. Find a +ve root of $f(x) = x^3 - 5x + 3 = 0$, using this method
5. Solve for a positive root of x-cos x = 0 by Regula Falsi method
6. Using Newton 's method ,Find a positive root of $xe^x = 1$, correct to four decimal places
7. Find the root between 1 and 2 of $2x^3 - 3x - 6 = 0$, by Newton-Raphson method correct to five decimal places.
8. Solve by gauss Seidel method x+y+54z=110, 27x+6y-z=85 ,6x+15y+2z=72 correct to three decimal places

9. Solve by Gauss-Seidel method, Solve 8x+y+z=8 ; 2x+4y+z=4; x+3y+5z=5
10. Using Gauss-Seidel method, Solve 10x-5y-2z=3: 4x-10y+3z=-3; x+6y+10z=-3 correct to three decimals
11. Solve the following system of equations, by using gauss Seidal method.

    28x+4y-z=32,x+3y+10z=24 , 2x +17y +4z =35.
12. Solve the following system of equations by Gauss Seidel's iteration method.

    $10x_1+2x_2+x_3=9$; $x_1+10x_2-x_3=-22$ $-2x_1+3x_2+10x_3=22$
13. Solve by Gauss Jordan method 10x+y+z=12, 2x+10y+z=13 ,x+y+5z=7
14. Solve by Gauss-Jordan method, the following systems of equations:

    x+5y+z = 14, 2x+y+3z = 13, 3x + y +4z = 17
15. Apply Gauss Jordan method to find the solution of the foll0wing 4x+2y+z=14, x+5y-z=10 ,x+y+8z=20
16. Find the inverse of the matrix $A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & -1 & 4 \\ -2 & 2 & 0 \end{bmatrix}$ by Gauss Jordan method
17. Find the inverse of the matrix $\begin{bmatrix} 3 & 1 & 2 \\ 2 & -3 & -1 \\ 1 & 2 & 1 \end{bmatrix}$ using Gauss-Jordan method
18. Find the numerically largest eigen values and the corresponding eigen vector using power method, given $\begin{bmatrix} 5 & 4 & 3 \\ 10 & 8 & 6 \\ 20 & -4 & 22 \end{bmatrix}$ starting vector is (1,1,1)
19. Find the numerically largest eigen value of $A = \begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{bmatrix}$ and the corresponding eigen vector by Power method
20. Find the numerically largest eigen value of $A = \begin{bmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{bmatrix}$ corresponding by Power method.

UNIT -2 INTERPOLATION AND APPROXIMATION

PART A

1. Write down Lagrangian polynomial for the data

   x: x_0 x_1 x_2

   y: y_0 y_1 y_2

2. Explain briefly interpolation.
3. **3.** Write down the Stirlings interpolation formula
4. **4.** State Newton's divided difference interpolation formula for unequal intervals
5. Find the third order difference using Newton's divided differences for the data

X	2	5	7	8
Y	1	2	3	4

6. Form the divided difference table for

   x: -1 1 2 4

   y: -1 5 23 119

7. What is the nature fo n^{th} divided differences of a polynomial of n^{th} degree?
8. Show that the divided difference is symmetrical in their arguments.
9. **9.** State Newton's forward interpolation formula
10. Derive Newton's backward difference formula.
11. Find the second divided differences with arguments a,b,c if f(x)=1/x.
12. Define forward , backward, central differences and divided difference
13. Using leg range's interpolation formula, find the value of y(10) from the table
    a. below

X	5	6	9	11
Y	12	13	14	16

PART-B

1) .Using Lagrange's formula, fit a polynomial to the data:

x: 0 1 3 4

y: -12 0 6 12

2.Fit a Lagrangian interpolating polynomial y=f(x) and find f(5).

X	1	3	4	6
Y	-3	0	30	132

3.Determine by Lagrange's interpolation method the percentage number of patients over 40 years using the following data:

Age over (x) years:	30	35	45	55
% number (y)of patients :	148	96	68	34

4.Using Newton's divided difference interpolation , find the polynomial of the given data

x:	-1	0	1	3
f:	2	1	0	-1

5.Find the value of y at x=28 from the following data

x:	20	23	26	29
y:	0.3420	0.3907	0.4384	0.4848

6.Find a polynomial of degree 4, which takes the following values, using Newton's Forward difference formula.

x:	2	4	6	8	10
y:	0	0	1	0	0

7. Find x when y=20 using Lagrange's interpolation formula for the data:

X	1	2	3	4
Y	1	8	27	64

8.Find the cubic spline interpolation.

x:	1	2	3	4	5
f:	1	0	1	0	1

9.Fit a Cubic spline for the following data:

X	0	1	2	3
Y	1	4	0	-2

10.Using cubic spline approximation, Find y(0.5) and y'(1) given M0=M2=0 and

X	0	1	2
Y	-5	-4	3

11.Find the cubic spline approximation for the function given below ,assuming that y''(1)=0 and y''(3)=0.

x:	1	2	3
y:	-8	-1	18

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12..The following data are taken from the steam table

Temp c	140	150	160	170	180
Pressure kg f/cm^2	3.685	4.854	6.302	8.076	10.225

Find the pressure at temperature t=175 C, using Newton's backward difference formula.

13.. Find y(12) using Newton's forward difference formula given:

X	10	20	30	40	50
Y	46	66	81	93	101

14.Using Lagrange's interpolation formula ,find y(10) from the following table

X	5	6	9	11
Y	12	13	14	16

15.From the following table of half-yearly premium for policies maturing at different ages , estimate the premium for policies maturing at age 46.

X:Age	45	50	55	60	65
Y:Primium	114.84	96.16	83.32	74.48	68.48

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UNIT -3-Numerical Differentiation & Integration

PART –A

1. State Romberg's method integration formula tom find the value of $I = \int_a^b f(x)\, dx$, using h and h/2.

2.Write down the simpson's 3/8 rule of integration given (n+1) data

3. What are the two types of errors involving in the numerical computation of derivatives?

4. Write down the trapezoidal and Simpson's rule for the evaluation of a double integral.

5. State the formula for trapazoidal rule of integration.

6. State the formula for 2 –point Gaussian quadrature.

7. In numerical integration , what should be the number of intervals to apply Simpson's one third rule and Simpson's three- eight rule.

8. Show that the divided difference operator Δ is linear

9. Write down the three point Gaussian quartered formula

10. Evaluate $\int_0^1 \frac{dx}{1+x^2}$, using Trapazoidal rule witth h = 0.2. Hence obtain an approximate value of π.

11. Evaluate $\int_{-1}^1 \frac{1}{1+x^4} dx$, using 2 –point Gaussian quadrature

12. Write 3/8 rule , assuming 3n intervals.

13. What are the truncation errors intrapazoidal rule and simpson's 1/3 rule

PART-B

1.Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at x = 10 for the following data

x	2	4	6	8	10
y	6	54	134	246	390

2.Evaluate $\int_0^6 \frac{dx}{1+x^3}$ by using trapezoidal and Simpson's 3/8 rule

3. Evaluate $\int_0^1 \frac{dx}{1+x}$ correct to three decimal places using rombergs method

4. Evaluate $\int_{-1}^1 \frac{dx}{1+x^4}$ by using 2 point Gaussian qudrature formula

5. Given that

X	1.0	1.1	1.2	1.3	1.4	1.5	1.6
Y	7.989	8.403	8.781	9.129	9.451	9.750	10.031

6.Find $\frac{dy}{dx}$ *and* $\frac{d^2y}{dx^2}$ at x = 1.1 and dy/dx at x = 1.

7.. Evaluate $\int_{2}^{2.4}\int_{4}^{4.4} xy\,dx\,dy$ using trapezoidal and Simpson's rule, dividing the range of x and y into equal parts.

8.Find f' (6) and the maximum value of y = f (x) given the data :

x :	0	2	3	4	7	9
f(x):	4	26	58	112	466	992

9.Evaluate $\int_{0.2}^{1.4} (\sin x - \log x + e^x)dx$,taking h = 0.2 by using

(a) Trapezoidal rule (b) Simpson's 1/3 rule

10 Evaluate, $\int_0^1 dx/(1+x^2)$ using Gauss three point formula

11.Use Trapezoidal rule to evaluate I = $\int_1^2 \int_1^2 dxdy/(x+y)$ taking h = k = 0.25

UNIT -4-Initial Value Problem for ODE

PART A

1. State the Taylor's series formula to find $y(x_1)$ for solving $\frac{dy}{dx} = f(x,y), y(x_0) = y_0$

2. Write down the demerits of Taylor's series method to solve the differential equations numerically.

3. Find the taylor's series up to x3 term satisfying 2y' + y = x + 1, y(0) = 1.

4. Find y(0.1) given y' = 1/2(x+y), y(0) = 1 by Euler method.

5. Using Eulers method find y(1.25) if $\frac{dy}{dx} = x^2 + y^2, y(1) = 1$, taking h = 0.25.

6. Using Eulers method , find y at x = 0.1 if $\frac{dy}{dx} = 1 + xy, y(0) = 2$.

7. Using modified Euler method, evaluate y(1.1) if $\frac{dy}{dx} + \frac{y}{x} = \frac{1}{x^2}, y(1) = 1$

8. What is meant by initial value problem and give an example for it.

9.State Milnes Predictor- Corrector formula.

10. Write Adam's Predictor- Corrector formula

11. Write down the Milnes Predictor- Corrector algorithms

PART-B

1.By means of Taylor series expansion, find y at x=0.1 and x=0.2 correct to three decimal places, given $\frac{dy}{dx} - 2y = 3e^x, y(0) = 0$

2Using Taylors series method, find y at x=0.1,0.2 given $\frac{dy}{dx} = x^2 - y$, y(0)=1 (correct to 4 decimal places)

3.Given $y` = x(x^2 + y^2)e^{-x}, y(0) = 1$ find y at x=0.1,0.2 and 0.3 by Taylor series method and compute y(0.4) by Milne`s method.

4. Find y(0.25) and y(0.5), using modified Euler`s method
With h=0.25, given that y`=3x ^{2}+y,y(0)=4. Compare the values with the exact solutions.

5.Compute y at x=0.25 by modified Euler`s method , given dy/dx=2xy, y(0)=1

6. Using modified Euler`s method, find y(0.1) and y(0.2) given dy/dx=x^2 + y^2 ,
y(0)=1 with h=0.1

7.Evaluate y(0.4) using Adam`s method, given $\frac{dy}{dx} = x^2 - y$, given y(0)=1,y(0.1)=0.9052,y(0.2)=0.8213 and y(0.3)=0.7492.

8.Given y`=1-y,y(0)=0 and y(0.1)=0.1. Obtain y(0.2) by improved Euler method and y(0.3) by Runge Kutta Fourth order method. Hence find y(0.4) by Milne`s method.

9.Apply fourth order Runge kutta method to find an approximate value of y when x=0.2, given that $\frac{dy}{dx} = x + y^2$, y=1 when x=0.

10.Given $\frac{dy}{dx} = x^2(1+y)$ and y(1)=1,y(1.1)=1.233,y(1.2)=1.548,
y(1.3)=1.979, evaluate y(1.4) by Milne`s predictor corrector method.

11.Find y(0.1) and y(0.2) using Runge Kutta Fourth order method from y`=x+y,y(0)=1.

12. Use R-K fourth order method to find y(0.2) for the equation

y`` - xy`+y=0, given that y(0)=1 and y`(0)=0.Take h=0.2.

13.Find y(0.1),y(0.2) and y(0.3) from $\frac{dy}{dx} = xy + y^2$, y(0)=1 by using Runge - kutta fourth order method, correct to 4 decimals.

14.Solve $\frac{dy}{dx} = y - x^2$, y(0)=1 (i) Find y(0.1) and y(0.2) by Runge - kutta fourth order method.(ii) Find y(0.3) by Euler`s method.(iii) Find y(0.4) by Milne`s predictor corrector method .

15.Solve y``-0.1(1- y^2)y`+y=0 subject to y(0)=0, y`(0)=1 using Runge - kutta fourth order method. Find y(0.2) and y`(0.2). Use step size Δx=0.2

16. Using Runge - kutta fourth order method, find y(0.8) correct to 4 decimals if y`=y-x^2 given y(0.6)=1.7379

17. Given dy/dx = ½(1+x^2)y^2 and y(0)=1, y(0.1)=1.06,y(0.2)=1.12,y(0.3)=1.21,evaluate y(0.4) by Milne`s predictor – corrector method.

18.Using Runge – Kutta method of fourth order, solve $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}, y(0) = 1$ at x=0.2 and x=0.4

19.Solve dy/dx=$\log_{10}$(x+y),y(0)=2 Using modified Euler`s method and find the values of y(0.2),y(0.4) and y(0.6), taking h=0.2

20. Given y``+xy`+y=0,y(0)=1,y`(0)=0,find the value of y(0.1) by Runge – Kutta method of fourth order.

21.Given $\frac{dy}{dx} = xy + y^2, y(0) = 1,$ y(0.1)=1.1169,y(0.2)=1.2773, find y(0.3) by Runge – Kutta method of fourth order and y(0.4) using Milne`s predictor-corrector method.

22.Solve the equation $\frac{dy}{dx} = 1 - y, y(0) = 0,$ using modified Euler`s method and compute y at x=0.1,0.2,0.3

23.Using Runge – Kutta method of fourth order, find y(0.8) correct to 4 decimal places if $y` = y - x^2, y(0.6) = 1.7379$ with h=0.1

24.Solve $\frac{dy}{dx} = xy + y^2, y(0) = 1,$ Using Milne`s predictor-corrector formulae and use Taylor series method to find y(0.1) , y(0.2) and y(0.3).

25.Apply Milne`s method , to find a solution of the differential equation $\frac{dy}{dx} = x - y^2$ at x = 0.8 , given the values.

X	0	0.2	0.4	0.6
Y	0	0.02	0.0795	0.1762

26.Using Milne`s predictor corrector method, find y(0.2) given y`=xy(1+y)
Given y(-0.2)= 1.0412,y(0)=1 and y(0.1)=1.0108 and y(-0.1)=1.0108

UNIT 5-Boundary Value Problems in ODE & PDE

PART A

1.For what points of x and y , the equations x .f_{xx} + y. f_{yy} = 0 , x.> 0, y>0 is elliptic

2. Define difference Quotient of a function y(x).
3. Derive Crank- Nicholson Scheme.
4. Write SFPF and DFPF formula used in solving Laplace equation $u_{xx} + u_{yy} = 0$ at the point (iΔx , iΔ**y)**
5. Write down the Crank-Nicholson scheme for solving one dimensional heat equation.
6. Express $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$ in terms of difference quotients
7. Write down the explicit formula tom solve one dimensional wave equations
8. Write down the finite difference scheme for solving y'' + x + y = 0,
   y(0) = y(1) = 0.
9. Write down the explicit formula to solving the hyperbola equations $\frac{\partial^2 u}{\partial t^2} = 9\frac{\partial^2 u}{\partial x^2}$ when Δx = h = ¼ and k = Δt = 1/16.
10. Define the local truncation error.
11. Write down the SFPF formula to solve Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$
12. Name at least two numerical methods that are used to solve one dimensional diffusion equation.

PART B

1. Solve the boundary value problem : $\Delta^2 u = -10(x^2 + y^2 + 10)$
   $xy'' + y = 0, y(1) = 1, y(2) = 2$, Using finite difference method by taking 4 intervals.

2.Solve the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ subject to the conditions u(x, 0) = sin π x, $0 \le x \le 1$; u(0,t) = u(l,t)=0 using crank –Nicolson method. Carryoutconsumptions for two levels, taking h =1/3, k = 1/36.

Evaluate the pivotal values of the equations $\frac{\partial^2 u}{\partial t^2} = 16\frac{\partial^2 u}{\partial x^2}$, taking h =1 up to t=1. The boundary conditions are u(0,t) = u(5,t) = 0 $\frac{\partial u}{\partial t_{(x,0)}} = 0$ and u(x,0) = $x^2(5-x)$

4. Solve the equations $\Delta^2 u = -10(x^2 + y^2 + 10)$ over the square mesh with sides x = y = 0, x =y =3 with u = 0 on the boundary and mesh length is 1.

5. Solve y'' –xy = 0 given y(0) = - 1 ; y(1) = 2 by finite difference method taking n = 2.

6. Solve $25u_{xx} = u_{tt}$ for u at the pivotal points given u(0,t) = u(5,t) =0, $u_t(x,0) = 0$ and u (x, 0) = 2x for $0 \le x \le 2$
   = 10-2x for $3 \le x \le 5$ up to t = 1 seconds taking h =1 and k=1/5.

7. Derive bender –schmidt recurrence formula to solve one dimensional heat equation.

8. Solve $\Delta^2 u = 0$ in the sqare region bounded by x = 0, x = 4, y = 0, y = 4 and with boundary conditions u(0,y) = y^2 / 2, u(4, y) = y^2, u(x,0) = 0, u(x, 0)=0 and u(x, 4) = 8+ 2x taking h=1 , k=1 (perform four iterations).

9. Solve $u_{xx} = u_{tt}$ for u at the pivotal points given u(0,t) = u(4, t) =0, $u_t(x,0) = x(4-x)/10$ and u (x, 0) = 0up to 4 time steps in t direction. Taking h =1 and k=1.

10. Solve $4\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$ with boundary conditions u(0,t) = u(8, t) =0, $u\ (x,0) = x(8-x)/2$ up to t = ½, taking h =1 and k=1/8.

11. Using Bender Schmidt recurrence method, solve numerically the parapolic equation $2u_{xx} = u_t$, subject to the conditions i) u(0,t) = 0 , t.> 0
ii) u(12,t) = 0 , t.> 0

iii) u(x,0) = 3x(12-x), $0 \le x \le 12$. Assuming h = 2 , Find the values of u up to t = 5 , properly choosing the step size k in the time direction

12. Solve the boundary value problem $x^2y'' - 2y + x = 0$, subject to the conditions y(2) =y(3) = 0.Find y(2.25), y(2.5) , y(2.75)

13. Solve the vibration problem $\frac{\partial u}{\partial t} = 4\frac{\partial^2 u}{\partial x^2}$ subject to the conditions y(0,t) = 0, y(8,t) = 0 y(x,0) =(1/2) x(8-x). Find y at x = 0, 2, 4, 6. Choosing $\Delta x = 2$, $\Delta t = ½$. Compute up to 4 time steps.

14. Solve $\nabla^2 u = -4(x+y)$ in the region given $0 \le x \le 4$, $0 \le y \le 4$, with all boundaries kept at 0• and choosing $\Delta x = 1$, $\Delta y = 1$. Start with zero vector and do 4 Gauss –seidal iterations.

15. Solve the equation $u_t = u_{xx}$,$0 \le x \le 4$, t > 0 subject to the condition to u(0,t) =0, u(4,t)=0 and u(x.0) = $(x/3)(16-x^2)$. By crank Nicholson method with h=k=1

16.Solve the equation $u_t = 4u_{xx}, 0 \le x \le 10$, t > 0 subject to the condition to u(0,t) =0, $\frac{\partial u}{\partial t}$ x,0 = 0 and u(x.0) = (x/100)(10-x), $0 \le x \le 10$. Compute u for three timesteps.

17.Solve the Laplace equation $u_{xx} + u_{yy} = 0$ for the square mesh with the boundary vaues, using Leibmanns method correct to integers.

18.Solve the boundary value problem $\frac{\partial y}{\partial t} = \frac{\partial^2 y}{\partial x^2}$, subject to the condition to u(0,t) =0, u(1,t)=0 and u(x.0) = sinπx, $0 \le x \le 1$.Using crank- nichalson method taking h =1/3, k = 1/36 with one time step.

19.Solve the b oundary value problem $x\ y'' + y = 0$, subject to the condition to y(1) = 1, y(2)=2 with h = 0.25by finite difference method

20. Solve the Laplace equation $u_{xx} + u_{yy} = 0$ for the square mesh with the boundary vaues as shown in the figure below.

21.Solve the equation y'' = x+y with boundary conditions y(0)= y(1)=0 numerically taking Δx = 0.25

22.Solve $u_{xx} = 32u_t$ with h = 0.25 for t>0, 0<x<1 and u(x,0)=u(0,t)=0, u(1,t) = t , using Bender-Schmit formula

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